## CAPACITOR-VALUE CALCULATION IN A THREE-PHASE ELECTRONIC CIRCUIT WITH THE HELP OF COMPLEX NUMBERS

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## Absztrakt

KONDENZÁTOR-ÉRTÉK SZÁMÍTÁS HÁROMFÁZISÚ ELEKTRONIKUS ÁRAMKÖRBEN ÖSSZETETT SZÁMOK SEGÍTSÉGÉVEL

Jelen tanulmány a komplex számok segítségével feltárja annak a kondenzátornak a megfelelő értékét, amely szükséges ahhoz, hogy egy háromfázisú generátortól egy családi házig és egy elektromos autó töltőállomásáig hatékonyabb legyen az energiaátvitel.

Kulcsszavak: komplex számok, környezetvédelem, elektrotechnika Diszciplina: matematika, fizika

## Abstract

This study uses complex numbers to reveal the appropriate value of the capacitor needed to efficiently transfer energy from a three-phase generator to a family home and a charging station of an electric car.

**Keywords:** complex numbers, environmental protection, electrical engineering **Discipline:** mathematics, physics

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## Introduction

Climate change and global warming mean a huge threat to the environment which can be reduced by implementing renewable and eco-friendly solutions in our everyday life this way minimizing our greenhouse gas emissions. Solar panels and wind turbines became more and more common in the last few years not only for supplying great factories or farms but also for family houses. In this exploration, we intend to calculate the correct value of the capacitor, the electronic device which makes a system that transmits energy more effective, for a family which aims to provide a sufficient amount of energy for their household and the charging of their electronic vehicle with an efficiency of 90% (I1).

Nowadays, if somebody invested in a renewable energy source for a house and with this solution, they are also able to keep track of the energy consumption and what amount of energy is produced by the generator and used by their home. By constantly checking the produced and consumed energy, it can be realized that a huge amount of it is wasted because of the inefficiency of the system. After a bit of research and communication with the manufacturer, we can come to the conclusion that we have to build a capacitor in the system in order to maximize efficiency and reduce wasted energy. An electrical capacitor is able to store electricity or electrical energy this way improving the efficiency of an AC (Alternating Current) circuit (Morton, 2014), however, the right value of the capacitor is dependent on several factors, so in this exploration we will try to determine the appropriate value of the capacitor that is required to make the energy transmission from a three-phase generator to a family house and to the charging station of an electric car more effective.

#### Methodology

At first, when I decided to calculate the value of the necessary capacitor for our system, I looked up the formula that was required and I found that there are several ones. The first one which I tried to interpret was this one (I2):

$$C = \frac{Qc}{\omega V^{2}}$$

And next to this formula Qc was also explained as:

$$Qc = \frac{Xc}{3}$$

Even though I understood that V is the voltage and that omega can be calculated as:

$$\omega = 2\pi f$$

where if is the frequency, I did not know what we mean by Xc, so I tried to find a video that explains it, and I found out that it is the resistance of the capacitor, and that it is the ratio between the effective voltage and the effective current in the circuit (I3). I found that reactive capacitance can be calculated as (I4):

## $Xc = P(TOTAL) (tan\alpha - tan\beta)$

Where  $\alpha$  is the phase angle of the current, in the beginning,  $\beta$  is the phase angle after applying the capacitor and Ptotal is the total power in the system. Because of my lack of knowledge in physics, I wasn't sure what phase angle means, so I decided to conduct some research before starting counting with it, and I found that the phase angle represents the phase difference between the current and the voltage, since they can be written as a function of sine (I5). The phase angle is basically the value of the shift applied either on the current this way getting the voltage or backwards.

So for calculating the reactive capacitance, I have to find the total power in the system and the phase angle of the current. I decided to start by exploring how I can calculate the phase angle of the current. In order to find this angle, it is not enough to find the expression for the current in one circuit, I have to find in both of them and then add them up (I6). For calculating the line current (current in one circuit) I found the following formula (I7):

$$I(line) = \frac{S}{\sqrt{3}V}$$

Where V is the voltage in the system and S is the Apparent Power, which will be explained later. I remembered from physics lessons that the notation for power was supposed to be "P", so I got a bit suspicious that this form is not correct, so I started to look for other ones. While watching a video (I8) about power and calculating the power transfer of a circuit I had to realize that there are three types of power:

1. Q - Reactive Power which can be calculated as follows:

$$Q = S \cdot sin\theta$$
 (19)

Where S is the Apparent power, which will be explained later, and  $\theta$  can be derived from the power factor. The definition of power factor was not clear for me, so I searched on it and found that the power factor (PF) is between 0 and 1 and if it is smaller than a unit it means that the voltage and the currents are not in phase, so they peak at different times (If we model them as functions of sine, their maximum occurs at different values). So, the power factor represents the difference in the phase of the voltage and current and can be calculated as (I10):

$$PF = cos\theta$$

The closer the power factor to one, the more efficient the system is, so I realized that during my calculations, I have to aim to modify the phase between the current and the voltage with the help of capacitors in order to achieve a power factor close to 1, preferably 0.9.

2. P - True Power and it can be calculated by multiplying the Apparent Power with the power factor:

$$P = S \cdot PF \quad (19)$$
$$P = S \cdot cos\theta \quad (19)$$

3. S - Apparent Power that can be calculated by using the formula:

 $S = P + jQ \quad (19)$ 

Where "j" is a complex number. At first, I couldn't understand why they use "j" to represent complex numbers, however, later one of my friends explained to me that in physics we use "j" instead of "i" because "i" also means current, however, in my calculations, I will use "i" as it is the appropriate notation for complex numbers in mathematics.

Since I realized that I will have to conduct operations with complex numbers I decided to search for the rules I have to keep in mind and I explored that complex numbers can be written in different formats that we have not yet covered in school (I11):

Complex form: Z = a + bi (we used previously)

Modulus-argument (polar) form:  $Z = r(cos\theta + isin\theta)$  (we not yet covered)

Exponential or Euler form:  $z = re^{-i\theta}$  (we not yet covered)

where r is the length of vector z and  $\theta$  is the angle between the real axis and the vector.

For later use I searched on how can I do different operations with the complex form and I found:

Addition: 
$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
  
Subtraction:  $(a + bi) - (c + di) = (a - c) + (b - d)i$   
Multiplication:  $(a + bi)(c + di) = (ca - bd) + (ad + bc)i$   
Division:  $\frac{a+bi}{c+di} = (a + bi)\frac{(c-di)}{c^2+d^2}$ 

The same operations in terms of the modulus argument forms:

Multiplication:

 $r1(\cos\theta 1 + isin\theta 1) * r2(\cos\theta 2 + isin\theta 2) = r1r2(\cos(\theta 1 + \theta 2) + isin(\theta 1 + \theta 2))$ Division:

$$\frac{r_{1}(\cos\theta_{1}+i\sin\theta_{2})}{r_{2}(\cos\theta_{2}+i\sin\theta_{2})}=\frac{r_{1}}{r_{2}}(\cos(\theta_{1}-\theta_{2})+i\sin(\theta_{1}-\theta_{2}))$$

However, I haven't found anything for addition and subtraction in modulus argument form, as in every case, when people wanted to add or subtract complex numbers that are in modulus argument form, they converted the number into its complex format.

In the case of the Euler form since it is only an exponential form, the operations work the same way if I would replace the letters with numbers, so:

$$r_{1}e^{i\theta_{1}} \cdot r_{2}e^{i\theta_{2}} = (r_{1} \cdot r_{2})^{i(\theta_{1}+\theta_{2})}$$
$$\frac{r_{1}e^{i\theta_{1}}}{r_{2}e^{i\theta_{2}}} = \left(\frac{r_{1}}{r_{2}}\right)e^{i(\theta_{1}-\theta_{2})}$$

However, in this case, addition and subtraction are not possible either, so during my calculations, I have to keep in mind that if I want to add up or subtract complex numbers, I always have to convert them to their complex form.

# Characteristics of the System Wind Turbines:

Now that I knew the formulas for my calculations, I started to search for the components that are given and the first one was the amount of transferred power. I found that the energy production of an onshore wind turbine can range from 250 watts to 7 MW, however, their output highly depends on the wind's speed through the rotor, so in calculations, we usually take only 25% of the maximum power output (I12). For this calculation the

turbine, EWT DW61 will be examined which has a maximum power output of 1MW (1000 kW), so a general output of 250 kW (I13).

It is important to mention that the used generator is a three-phase generator, as the formulas for determining the values of different components in the system are not the same as in the case of a single-phase system, and also because a capacitor is required for all the three circuit lines, so in the end, the calculated value has to be divide by three (I14).

Power requirements for charging an electric vehicle:

The second data I was sure I could find was the power requirements of the two systems I want to supply, so I also searched for them. As the basis of my calculation, the EV Nissan Leaf will be used as this is the car that my family wants to charge next to supply our house with electricity and also because this is one of the most common EVs on the market with its relatively cheap price compared to the other ones, so these calculations may be helpful for other households as well. The Standard Leaf model is equipped with a 40kWh battery pack (I15) and according to Source chargedevs.com an average EV without power factor compensation with a charger that is connected to a three-phase generator has a 0.6 PF but can be increased to 0.9 with an appropriate sized capacitor (Jeffrey, 2018).

Family house:

The next step was to get the energy consumption of a home, however, this number is not steady and ranges from 20 to 45 kWh per day. I decided to calculate with an average number. In the US the average annual electricity consumption is 11000 kWh, which is approximately 37 kWh per day (I16), and since our consumption is also around this number, I think 37 kWh can be considered as the rough average for our consumption as well, and by using this number, my calculations could also be implemented for other households.

I also had to realize that in the case of a family house, it is more difficult to calculate the power factor because several electronic devices are supplied by the circuit. However, I found that most motor-driven appliances in a household have a power factor of 0.75, so in this research, this number will be considered (I17).

### Calculations

Visualizing the problems that were to be solved always helped me to start my calculations, so I decided to try to model the problem. The two loads, the family house and the charger of the EV are connected to the three-phase circuit as depicted in Figure 1.

Figure 1: Representation Of The Two Loads.



the total power of the two loads, for which coordinate system to help visualize the I have to add up the real power in the two circuits.

 $S_{TOTAL} = S_1 + S_2$ 

As I mentioned before,  $S_1$  and  $S_2$  can be calculated as:

 $S_n = P_n + iQ$ 

From this formula I only know P, however, I realized that from the formula of P, I know P and the power factor, so the only missing component is S, so first I will calculate S1.

$$P = S \cdot PF$$
37 kW = S1 \cdot 0.75
  
S1 =  $\frac{37}{0.75}$  = 49.33333 kVA \approx 49.33kVA |

The other component I am looking for is Q, so by substituting back to its formula I can calculate it:

```
0 = S \cdot sin\theta
Where
          \theta = \arccos PF (because: PF = \cos\theta, and PF = 0.75)
 Q1 = 49.33 · sin (arccos 0.75) = 49.33 · 0.6614378278 = 32.62872804
                \approx 32.63 \, kVAR
As I know P1 and and Q1 now I can write an equation for S1:
                     S1 = (37 \, kW + i \, 32.63 \, kVAR) \, kVA
                           S1 = (37 + i32.63) kVA
```

Following the same method I can find the complex form for S<sub>2</sub>:

$$S2 = \frac{40}{0.6} = 66.6666 \approx 66.67 \ kVA$$

$$Q2 = 66.67 \ * \ sin(arccos 0.6) = 66.67 \ * \ 0.8 = 53.336 \approx 53.34 \ kVAR$$

$$S2 = (40 \ kW \ + \ i53.34 \ kVAR) \ kVA$$

$$S2 = (40 \ + \ i53.34) \ kVA$$

up  $S_1$  and  $S_2$  for which I decided to voltage, so I started to think about which

As I realized before, at first, I have to find represent the two vectors in the same problem (Figure 2).

> Figure 2: Representation of S1 and S2 Imaginary axis



As I mentioned above, if I want to add up complex numbers, I have to use the complex forms of them, so I did it in the following way:

S(TOTAL) = (37 + i 32.63) + (40 + i 53.34) = (77 + i 85.97) kVA

After calculating the total real power requirements of the system, I can start looking for the phase angle of the total current which can be calculated by adding up the two line currents in the two load circuits, so first, I will try to find I1 and I2. To determine the line currents, I will use the formula mentioned above:

$$In = \frac{Sn}{\sqrt{3} * V}$$

According to the formula, I have to divide In order to calculate S<sub>TOTAL</sub> I have to add the apparent power with root three and the format should be the most suitable for this operation. By looking at the operations provided above, I decided that the Euler form seems to be the most suitable to divide with, so I will use these forms. I searched on how could I convert the complex number I calculated for S1 to its modulus-argument and then exponential format, and I learned that if I draw S1 as a vector, the modulus of the vector is the length of it, which is represented with "r" in the formula, and  $\theta$  is the angle between the real axis and the vector (I18).

To better represent what do I have to calculate, I decided to draw a diagram and represent S1 as a vector (Figure 3).





As mentioned above, r is the modulus or length of the vector, which can be easily calculated by using the Pythagorean theorem:

Modulus 
$$(r)$$
 :  $|S1|$ 

$$=\sqrt{37}^{2}$$
 + 32.63<sup>2</sup>

Modulus (r) :  $|S1| = 49.3327632 \approx 49.33$ 

And the Argument part or,  $\theta$  can be calculated with trigonometry, by finding the angle between S1 and the real axis. To find this angle I will use the inverse of the tangent function, arctan:

Argument ( $\theta$ ): arctan  $\frac{32.63}{37}$ Argument ( $\theta$ ): 41.40880955°  $\approx$  41.41°

Now that I know r and  $\theta$ , I can substitute these values back to the modulus-argument form:

$$S1 = r(\cos\theta + i\sin\theta)$$
  
$$S1 = 49.33 (\cos 41.41^{\circ})$$

By using these numbers, I also can write the exponential (Euler) form:

$$S1 = re^{i\theta}$$
$$S1 = 49.33e^{i41.41^{\circ}}$$

S1 vector this way can be represented in a coordinate system (Figure 4).



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I also found that the converted formats can be represented in a polar coordinate system (see: Figure 5 - I19).





Using the same method, I also can calculate the modulus and argument parts of S2, to convert them to other formats. First I decided to represent S2 in a coordinate system (Figure 6).



Modulus (r) : |S2|=  $\sqrt{40^2 + 53.34^2}$ Modulus (r) : |S2| = 66.67200012  $\approx$  66.67

Argument (
$$\theta$$
): arctan  $\frac{53.54}{40}$   
Argument ( $\theta$ ): 53.13353983°  
 $\approx$  53.13°

Now by using r and  $\theta$ , I can write the equation of S2 in its modulus-argument and Euler format as well (Figure 7): Modulus-argument form:  $S2 = 66.67 (\cos 53.13^{\circ} + i \sin 53.13^{\circ}) kVA$ Exponential (Euler) form:  $S2 = 66.67 e^{i53.13} kVA$ 

Figure 7: Modulus Argument Representation S2



S2 can also be represented in a coordinate and a polar coordinate system (Figure 8).





I wasn't sure how to convert the source voltage into its exponential form, however, I found that since it is not shifted anywhere, it can be easily expressed as (I20):

## $V = 250000 e^{-i0} V$

Using this knowledge, now I can substitute to the formula to find I1 and I2:

$$I1 = \frac{49.33 e^{-i41.41}}{\sqrt{3} * 250000 e^{-i0}}$$
$$= 0.00011392275 e^{-i41.41}A$$

This number was suspiciously small for me, so after a bit of thinking, I realized that I have to convert the apparent power from kVA to VA, so I have to multiply the modulus part by 1000. I wasn't sure whether I can do something with the argument part as well, or not, but after watching a video that explains the parts, I realized that this part will remain the same, as it only represents the phase shift of the apparent power, which shift will remain the same even if I convert it to another measurement (I21). This way the corrected version can be written as:

$$I1 = \frac{49330 \ e^{-i41.41}}{\sqrt{3} \ * 250000 \ e^{-i0}}$$
  
= 0.1139227551e^{-i41.41} A  
= 113.9227551e^{-i41.41} mA \approx  
 $\approx 113.92e^{-i41.41} mA$   
$$I2 = \frac{66670 \ e^{-i53.13}}{\sqrt{3} \ * 250000 \ e^{-i0}}$$
  
= 0.1539677698e^{-i53.13} A  
= 153.9677698 \ e^{-i53.13} mA \approx  
 $\approx 153.97 \ e^{-i53.13} mA$ 

In order to find  $I_a I$  have to add up I1 and I2, however, for this, I have to convert them into a format that I can add up, because, in the case of the exponential forms it is not possible. As I expressed above, only the complex forms are suitable for addition, so now I will try to convert I1 and I2 to their complex forms by finding the true power and the reactive power using the Pythagorean identity and arcus- tangent as I earlier discovered, but now, backwards:

$$\sqrt{P^2 + Q^2} = 113.92$$
$$arctan\frac{Q}{P} = 41.41^{\circ}$$

Now I will attempt to solve it as an equation system: from the second equation I can express Q/P then Q by:

$$tan 41.41^{\circ} = \frac{Q}{P} = 0.881928829$$
  
 $Q \approx 0.88 P$ 

Now I can substitute P back to the first equation:

$$\sqrt{P^{2} + (0.88P)^{2}} = 113.92$$

$$P^{2} + 0.7744 * P^{2} = 12977.7664$$

$$P^{2} (1 + 0.7744) = 12977.7664$$

$$P^{2} = \frac{12977.7664}{1.7744} = 7313.889991$$
$$P = 85.52128385 \approx 85.52$$

Now I can substitute P to the second equation and try to express Q:

$$0.88 = \frac{Q}{85.52}$$

 $Q = 0.88 * 85.52 = 75.2576 \approx 75.26$ Thus I<sub>1</sub> can be expressed in its complex form as:

I1 = (85.52 + i75.26) mAI also have to find I<sub>2</sub>, for which I will follow the exact same methods:

$$\sqrt{P^{2} + Q^{2}} = 153.97$$

$$arctan \frac{Q}{P} = 53.13^{\circ}$$

$$tan 53.13^{\circ} = \frac{Q}{P} = 1.333328371$$

$$Q \approx 1.33 P$$

$$\sqrt{P^{2} + (1.33P)^{2}} = 153.97$$

$$P^{2} + 1.7689 * P^{2} = 23706.7609$$

$$P^{2} (1 + 1.7689) = 23706.7609$$

$$P^{2} = \frac{23706.7609}{2.7689} = 8561.797429$$

$$P = 92.52998124 \approx 92.53$$

$$1.33 = \frac{Q}{92.53}$$

$$Q = 1.33 * 92.53 = 123.0649$$

$$\approx 123.06$$

$$I2 = (92.53 + i123.06) mA$$

As now I know I1 and I2, I can calculate I<sub>a</sub> by adding them up:

$$Ia = (85.52 + i75.26) + (92.53 + i123.06)$$
$$= 178.05 + i198.32$$

From a video, I also learned that it is important whether the voltage lags the current or backwards, and in this case, since the complex conjugate of this number form of the capacitor's reactance: (I22):

$$Ia * = 178.05 - i198.32$$

To find the right value of the capacitor, we have to find the angle by which the current is translated, so we have to convert it to its modulus-argument form:

$$\theta = \arctan \frac{-198.32}{178.05}$$
  
= -48.08277632  
 $\approx -48.08$   
 $r = \sqrt{(178.05)^{-2} + (-198.05)^{-2}} =$   
266.3186156  $\approx 266.32$   
 $Ia = 266.32 e^{-i(-48.08)}$ 

Now that I know the angle I was thinking about what could be the next step. I decided to see what the power factor is, however, I realized that I won't need it in the future, but at least this number explains why our system is ineffective, as this number is very far from 1.

$$PF = cos (-48.08)$$
  
= 0.6680923285  
 $\approx 0.67$ 

I want something closer to 1, so now I can try to adopt a capacitor that will shift the angle to achieve a power factor of 0.9. Since in the formula of the capacitive reactance I need the phase angle to which I want to transfer the current and not the power factor, I decided to calculate the angle by using the formula:

$$\arccos 0.9 = \theta$$

## $\theta = 25.84^{\circ}$

So now, I know the current angle, the desired angle and also the total true power

the current lags the voltage, we have to take in the system, so I can substitute to the

$$Xc = Ptotal (tan48.08 - tan25.84)$$

$$=$$

$$= 77000 (1.113735859)$$

$$- 0.4842804597)$$

$$= 48468.06574$$

$$\approx 48468.07 VAR$$

As I mentioned in the first part, I can not forget that this value that I calculated is the total reactance for the three-phase, so now we have to divide it by 3:

$$Qc = \frac{48468.07}{3} = 16156.02333$$
  
\$\approx 16156.02 VAR\$

And to calculate the value of the capacitor, I have to substitute everything back to the original formula:

$$C = \frac{Qc}{\omega V^{2}}$$

$$C = \frac{16156.02}{2 \cdot \pi \cdot 50 \cdot 250000^{2}}$$

$$= 0.00822819342F$$

$$= 822.819342\mu F$$

$$\approx 822.82 \ \mu F$$

So the value of the capacitor that our system requires is 822.82  $\mu F$ .

I decided to search on what types of capacitors exist, and I found that the commonly available capacitor value is rather 680 or 1000  $\mu F$ , so there is room for later calculations on deciding which capacitor would be the best for our system

or we rather should apply a combination of different capacitors (I23).

## Conclusion

In conclusion, my calculations showed that if we want to supply a family house with energy which has а general consumption of 37kWh and a charging station of a Nissan Leaf which has a consumption of 40kWh and the power supply is an EWT DW61 type of wind turbine with a voltage output of 250kV with 50Hz frequency, then the value of the necessary capacitor is  $804.8385 \,\mu F$  to achieve a 0.9 Power Factor. Even though these values are taken from a family's example, this can be generalized, as a house's consumption is in line with the I4: average consumption and the Nissan Leaf is one of the most common electric cars on the market.

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